DAVID S. SCHWARZ & ASSOCIATES

Statistical Consultants

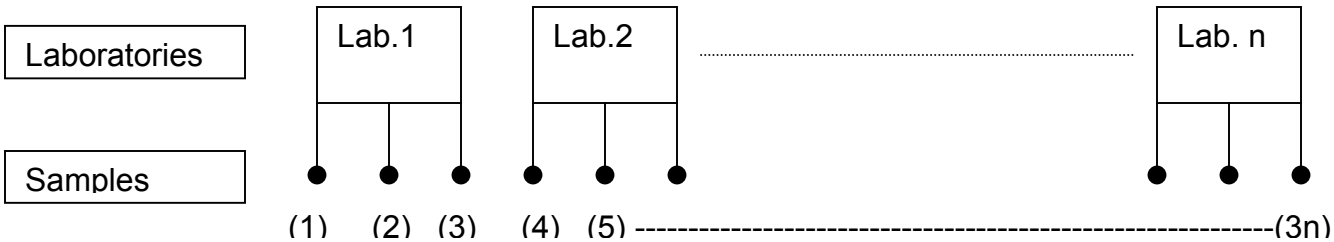
4261 Americana Drive
Suite 229
Stow, OH 44224

Phone: (330) 945-4733
Fax: (330) 945-4733
E-Mail: david.schwarz@pobox.com

Determination of the Total Standard Deviation of a Process Having Multiple Sources of Variability

Many processes have multiple sources of variability that need to be taken into account by one number or index. Consider the common example of a series of laboratories where a test is run using one test instrument. In this case, two sources of variability are present, 1) sample to sample differences obtained within a given laboratory and 2) differences between the laboratories themselves.

This situation may be pictured as follows:



As each lab. runs it's tests, differences between individual samples will be observed. This is simply due to natural variation. When all the data is collected, one will note that there are differences between labs. which can be attributed to the same natural variation except that a certain amount of it is partitioned off for each of the labs individually.

Therefore, one can say that the labs. make a unique, individual, contribution to the total variability that is independent of the variability contributed by the individual samples within each lab.

One can express this idea algebraically in the following way.

Define: S_T = The total standard deviation; a composite measure of variability.

S_S = The standard deviation associated with differences between individual samples within a lab. This is also the pooled standard deviation.

S_L = The standard deviation associated with differences between laboratories after S_S has been removed. (Calculation not shown here).

n = The number of samples tested per lab.

Then:

$$S_T = \sqrt{S_L^2 + \frac{S_S^2}{n}}$$

This principle of multiple sources can be extended beyond two by adding more layers of variability sources (such as plant, raw material supplier, etc.).